Multipath Flowrate Measurements of Symmetric and Asymmetric Flows

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Abstract *- The technique of ultrasonic flowrate measurements by multipath flow-cells is presented and analized. It is shown that accuracy of quadrature integration as a basic tool for data processing in ultrasonic multipath spoolpieces abruptly deteriorates in the presence of flow asymmetry. To obtain metrological effectiveness of the multipath flowcells for the case of disturbed flows, full characterization and reconstruction of the flow velocity profile in each point of some cross-section of the transport channel is necessary. As a first stage, an implementation of Abel's transform is suggested.*

1. INTRODUCTION

The evaluation of integral characteristics of flow velocity fields, in particular flowrate of a liquid or gas being transported through the channels of various cross-sections, represents actual theoretical, computing and engineering problem [1]. The appropriate methods of measurements and characterization of the flows can be subdivided on two groups: direct measurements and indirect those (remote sensing and interpretation of registered data). The first group includes various sorts of anemometric methods, vortex shedding devices, system of pressure measurements on a cross-section of the channel etc. The devices of the second group use in the basis effects of interaction of some test signal with a flow under the investigation. Thus it is assumed that such interaction does not carry a revolting character in relation to hydrodynamic characteristics of a flow being investigated.

The technique of ultrasonic time-of-flight flow measurements, optimization and increasing informative capacity of which is a purpose of the given paper, consists in registration of a transit-time difference of ultrasonic signals, generated by a pair of transducers, fixed on some distance from each other on walls of the measuring channel. At the presence of a flow, the time of first registration of an ultrasonic signal, propagating in a direction of a flow (downstream signal), is reduced in comparison with similar one for a pulse, generated in the opposite direction (upstream signal); the time-of-flight difference Δt being thus fixed is proportional to the mean value of a flow velocity vector across a propagating trajectory of the signal.

An accuracy of the flow velocity measurements achieved hitherto in practical (non-laboratory) operating conditions comes to $\pm$ 1÷2 % at the flow velocities > 0.3 m/s and $\pm$ 0.01 m/s at the velocities $\le$ 0.3 m/s. It is necessary to take into account, that the achievement of so high accuracy of measurements has arisen from the development of the high-precision technique for the registration of times-of-flight differences of acoustic signals as well as by design of special multisensor measuring flowcells [2], [3].

2. SPOOLPIECE'S DESIGN AND SCHEME OF MEASUREMENTS

Design of the flowcell represents a number of transducer's pairs being set into the module walls and forming appropriate number of measuring planes directed parallel to the axis of the channel and being away from the axis at the predefined distances (Fig. 1,2). The magnitudes being registered in measuring process are the time-of-flight differences of ultrasonic pulses generated in opposite directions by transceivers A_{n1} and A_{n2} and single reflected at the opposite wall of the module in points B_{n1}, where n is the number of measuring plane. The geometric configuration of measuring flowcell should ensure a maximum of SNR in frameworks of the chosen operating mode. With this purpose each transducing or receiving element should be oriented so that maximum of its directivity diagram has to be coincided with predicted direction of signal propagation. Secondly, with the purpose of eliminating the influences of wall's curvature in prospective points of signal reflection B_{nm} special reflective platforms oriented in a planes of measurements should be prepared. The index m

characterizes a prospective multiplicity of the signal reflections. With a purpose of simplification, in the Fig. 1 three measuring planes are only represented: two planes corresponding to positive chord values *x*, and diameter measuring plane. Design and equipment of multipath spoolpieces allow to use effective procedures of a numerical integration (quadrature formulae) which provide with an considerable rise of accuracy even in the case of very limited set of measuring data.

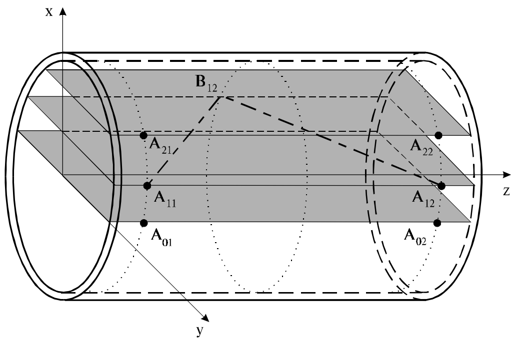

Figure 1. Scheme of multipath ultrasonic flowrate measurements in the standard GC-flowcell

Figure 2. General view of the measuring GC-spoolpiece. Panametrics, Inc. (1998), reproduced with permission

3. BASIC EVALUATION EQUATIONS

3.1. Approximation of an average velocity profile

Let $\boxtimes_n$ is a distance of *n* - th measuring plane from the origin of the co-ordinate system; *D* is an internal diameter of the channel and *L* is a distance between transmitting and receiving transducers. Let's consider vortex-free movement of the transported medium in the channel in direction of axis *z* , i.e., ***v*** = (0, 0, $v_z(x, y)$). Let's assume also, that the cross-sectional distribution of the velocity vector v_z in the channel has axial symmetry, and, hence, $v_z = v_z$ (❒).

Then, as it follows from geometry of the problem, the travel time of acoustic impulse generated by transmitting element A_{n1} and registered by receiving element A_{n2}, i.e., in direction along of stream, is equal to

$$T_{A_{n1} \to A_{n2}} = \frac{2\,|A_{n1}B_{n1}|}{c_0 + \dfrac{L}{2\,|A_{n1}B_{n1}|}\bar{v}_z(x_n)}, \quad (1)$$

where

$$\bar{v}_z(x_n) = \frac{1}{|A_{n1}B_{n1}|}\int_0^{|A_{n1}B_{n1}|} v_z(l)\,dl \qquad (2)$$

is an average flow velocity in *n* - th measuring section of the channel and integration is produced along the direction of propagation.

Similarly, the propagation time of acoustic impulse generated by transmitting element A_{n2} and registered by receiving element A_{n1}, i.e., in the direction, opposite to a stream, is equal to

$$T_{A_{n2} \to A_{n1}} = \frac{2\,|A_{n1}B_{n1}|}{c_0 - \dfrac{L}{2\,|A_{n1}B_{n1}|}\bar{v}_z(x_n)}. \quad (3)$$

It is obvious, that if $\bar{v}_z(x_n) \neq 0$, then $t_{A_{n2} \to A_{n1}} > t_{A_{n1} \to A_{n2}}$. The appropriate difference in travel times stipulated by the presence of flow in the measuring channel, $\Delta t(x_n) = t_{A_{n2} \to A_{n1}} - t_{A_{n1} \to A_{n2}}$, can be expressed as

$$\Delta t(x_n) = \frac{4\,|A_{n1}B_{n1}|\sin a}{c_0^2} v_z(x_n). \quad (4)$$

Thus, as follows from (4), the mean value of the flow velocity $\bar{v}_z(x_n)$ in *n* - th measuring section can be obtained basing on the measurements of $\Delta t(x_n)$ as

$$v_z(x_n) = \frac{c_0^2}{2L}\Delta t(x_n). \qquad (5)$$

3.2. Evaluation of the flowrate magnitude

A flowrate *Q* of a liquid or gas being transported in the pipeline is defined as an amount of a liquid or gas flowing through cros-ssection *S* of the transporting channel. For the stable stream, the flowrate is determined as

$$Q = \iint_S v_z(x,y)\,dx\,dy \ . \qquad (6)$$

The formula (6) can be rewritten as

$$Q = \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} v_z(x,y)\,dx\,dy \ . \qquad (7)$$

Replacing in (2) integration over *l* by an integration over *y*

$$\bar{v}_z(x_n) = \frac{1}{2\sqrt{R^2 - x_n^2}} \int_{-\sqrt{R^2-x_n^2}}^{\sqrt{R^2-x_n^2}} v_z(x_n, y)\,dy \qquad (8)$$

and substituting (8) in (7), we shall receive

$$Q = 2\int_{-R}^{R} \sqrt{R^2 - x^2}\,\bar{v}_z(x)\,dx \ , \qquad (9)$$

or

$$Q = \frac{c_0^2}{L} \int_{-R}^{R} \sqrt{R^2 - x^2}\,\Delta t(x)\,dx \ . \qquad (10)$$

Making the change of variables $x = Rt$ in (9) and (10), we get

$$Q = 2R^2 \int_{-1}^{+1} \sqrt{1-t^2}\,\bar{v}_z(Rt)\,dt \ , \qquad (11)$$

or

$$Q = \frac{R^2 c_0^2}{L} \int_{-1}^{+1} \sqrt{1-t^2}\,\Delta t(Rt)\,dt \ . \qquad (12)$$

As the measuring data $\Delta t(x) = \Delta t(Rt)$ as well as the magnitudes of mean values of the velocity $\bar{v}_z(x) = \bar{v}_z(Rt)$ are accessible in real conditions only in the limited set of measuring planes, for the evaluation *Q* with the help of (11) - (12) it is necessary to apply numerical integration, by which the specified integrals are replaced by the appropriate quadrature formulae [4].

If, for example, sampling points (positions of the measuring planes) are chosen in such a manner that

$$t_n = \cos\frac{np}{N+1} \ , \qquad (13)$$

then the combination of (11) and (13) leads to:

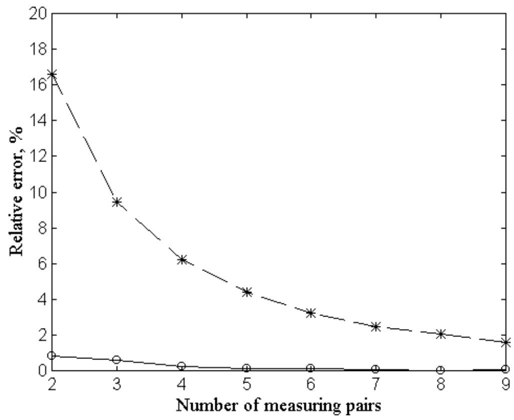

Figure 3. Relative errors of flowrate measurements for a test velocity profile calculated by ( ) - quadrature and ( ) - trapezoidal integration

$$Q = \frac{2pR^2}{N+1}\sum_{n=1}^{N}\sin^2\frac{pn}{N+1}\,\bar{v}_z(Rt_n) \ . \qquad (14)$$

Effectiveness of this approach for a test velocity profile having axial symmetry is demonstrated in Fig. 3.

The further increase of accuracy can be reached on a path of use together with the data obtained in basic parallel planes those obtained in additional, so-called "skewed" planes, which are not parallel to basic measuring planes.

4. MODELING OF ASYMMETRICALLY DISTORTED FLOW

The technique of direct integral estimations is valid in that case only if two-dimensional velocity profile in the cross-section has the radial symmetry. However, it is well known that most fluid flows in circular pipes are, in practice, asymmetric. So, for example, an existence of the bends in pipe systems leads to deformation of the axial velocity profiles, non axisymmetric flow behavior, etc. that reduce the performance of the multipath ultrasonic flowmeters [5],[6].

To demonstrate this fact, we will use for test numerical simulation a velocity profile determined in the form:

$$v_z(r,q) = \sin\frac{p}{2}(1-r)^{1/5} + a\sin p(1-r)^{1/2} e^{-0.2q}\sin q \ , \qquad (15)$$

what is quite similar to velocity profile after bends [7]. The value *a* in (15) is the coefficient of the asymmetry. Appropriate values of relative errors of the flowrate measurements calculated for the standard GC-spoolpiece having 4 measurement planes are presented in Table 1. Values

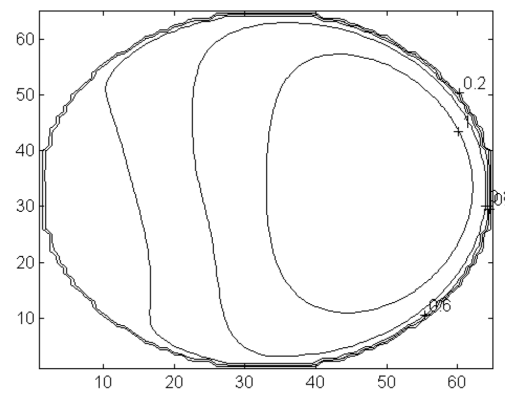

Figure 4. Contour graph of the local velocity calculated by using (15) for a=0.3

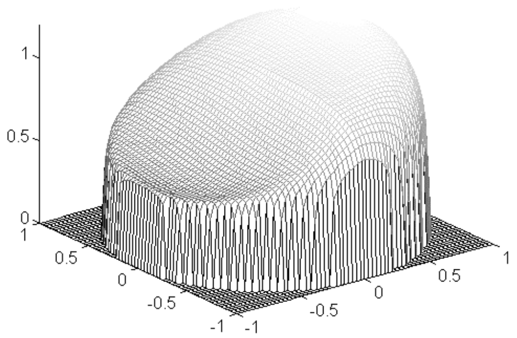

Figure 5. 3-D mesh of the velocity profile after the bend

s_t were computed by trapezoidal integration and s_q by Chebyshev quadrature integration.

It is necessary to point out that the results presented in Table 1 have an empirical nature and can not be interpreted by such a way that for asymmetric velocity profile trapezoidal integration gives better accuracy as quadrature one. The results demonstrate here the fact that performance of an ultrasonic multipath flowmeter system in real conditions depends on the path configuration as well as on the orientation of the paths with respect to the flow disturbances.

Table 1. Relative errors of flowrate measurements for asymmetric flow

a	$s_{t,\%}$	$s_{q,\%}$
0.0	5.9	0.5
0.1	7.3	7.7
0.2	7.7	15.1
0.3	14.8	22.8
0.4	22.3	30.7

In such situation, the unique correct way of precise estimating flowrate value is complete reconstruction of 2-D velocity field distribution by an algebraic reconstruction technique or some quasi-tomographic reconstruction method efficient in the case of incomplete data. The implementation one of them, Abel's transform based inversion, is demonstrated below.

5. INTEGRAL ABEL'S TRANSFORM

Result of ultrasonic travel time measurements is a mean integral value of the flow velocity along the way of ultrasonic signal. Using procedures of interpolation it is possible to receive a more exact structure of such average velocity in a pipe cross-section.

As was shown above, the measured average velocity $\bar{v}_z(x)$ is concerned with the unknown radial distribution $v_z(r)$ of the true flow velocity by a relation

$$\bar{v}_z(x) = \frac{1}{\sqrt{R^2 - x^2}} \int_{|x|}^{R} \frac{r v_z(r)\,dr}{\sqrt{r^2 - x^2}}, \quad (16)$$

where R is pipe's radius, x is a distance from an ultrasonic beam up to the axis of transporting channel.

The relation (16) is the modified integral transformation of Abel's type for function $v_z(r)$ [8],[9]. It is possible to receive the next formula of the inverse transformation for (16):

$$\bar{v}_z(r) = -\frac{2}{pr}\frac{d}{dr}\int_r^R x\bar{v}_z(x)\sqrt{\frac{R^2 - x^2}{r^2 - x^2}}\,dx\,. \quad (17)$$

In the case of numerical realization of (17) there is a number of problems concerned with singularity of the integrand at $x = r$ and $x = r = R$ as well as with a general incorrectness of the integral equations of the first kind [8]-[10]. As the accuracy of reconstruction depends on the number of sampling points, for the characteristic of a deviation of reconstructed function from initial function three criteria were chosen in three metrics - mean-square, absolute and maximal, accordingly:

$$\left.\begin{array}{l} D = \sqrt{\dfrac{1}{N}\sum\limits_{n=1}^{N}\left[v_z(r_n) - \tilde{v}_z(r_n)\right]^2} \\ R = \dfrac{1}{N}\sum\limits_{n=1}^{N}\left|v_z(r_n) - \tilde{v}_z(r_n)\right| \\ E = \max\limits_{n\in[1,N]}\left|v_z(r_n) - \tilde{v}_z(r_n)\right| \end{array}\right\}, \quad (18)$$

where $v_z(r_n)$ is given distribution of flow velocity at a sampling point r_n, $\tilde{v}_z(r_n)$ is reconstructed distribution of flow velocity at the same point, *N* is a number of sampling points.

In Fig. 6, an example of the initial and reconstructed profiles of original flow velocity is shown for a demonstration of the applicability of the inverse formula (17).

In Fig. 7, the dependencies of deviations of a reconstructed profile from true profile on the number of sampling points are shown. Applying powerful Abel's inversion method we have used the fact that the distortion of the velocity field structure occurs regularly, namely, the lines of the equal values representing a circles in a radial symmetry case, are deformed under the known law. Then the 2-D problem of a structure reconstruction becomes 1-D problem [9]. Thus, as a first step, there was natural to implement the complete reconstruction of a true velocity structure in the case of radial symmetry and to compare them with the standard integration technique.

Abel's reconstructive technique was demonstrated by inverting a simulated data and is shown to yielded results of a quality equal to that of a recent integral technique. It is of interest also to investigate the features of the inverse Abel's transformation use for a reconstruction of initial velocity field, in particular in the case of the limited number of the measured points. The following step could be a new way for a reconstruction of asymmetrical, but regularly deformed 2-D image of the structure of a velocity field.

In this paper we have considered only the problem of the reconstruction of the axial velocity profile. Some results dealing with the numerical inversion of the cross-flows can be found in [11],[12].

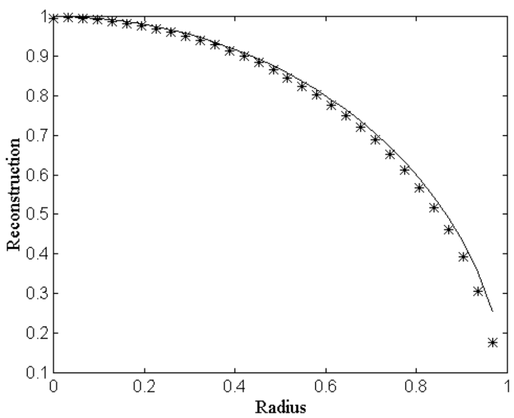

Figure 6. Initial (solid line) and reconstructed (□) profiles of flow velocity

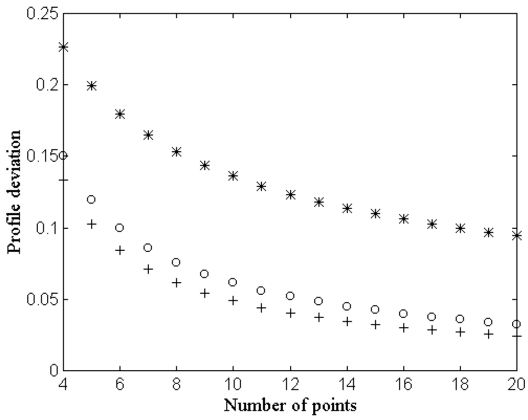

Figure 7. Dependencies of deviations of a reconstructed profile from given profile on the number of sampling points in three metrics: mean-square (□), absolute (+) and maximal (□)

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