Detection of Scattering and Absorption by NIR Tomography

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Abstract - *In this paper we present an iterative method for solving the inverse problem to reconstruct the distribution of both scattering and absorption simultaneously from data given by near infrared tomography.*

Keywords : Inverse Problems, Tomography, Diffusion approximation

1 INTRODUCTION

A possible application of near infrared (NIR) tomography is to get a spatial resolution of some optical properties of human tissue, i.e. scattering and absorption. It is known that these properties could be used to detect tumors at very early state if the image reconstruction method is sensitive enough.

The detection of both scattering and absorption simultaneously from data given by NIR tomography is essential for the medical purpose of this noninvasive technique since only then a reliable distinction between malicious and healthy tissue can be made. In this paper we present a simple and effective iterative method which could be thought of as prototype for this purpose. The plan of the paper is as follows:

In section 2 the physical background is given that will serve as mathematical basis for the inverse problem posed by the tomographic context.

Section 3 describes the formulas calculating the boundary data in order to obtain a boundary value problem as forward model for the iterative method which will successively approximate a minimum norm solution of the tomographic problem.

In section 4 the iterative method is developed as a nonlinear extension of the classical method of Karzmarz for linear problems. A pseudo code of the algorithm is given.

Finally, in section 5 results of our method for some artificial phantoms are presented.

2 PHYSICAL BACKGROUND

We suppose that the optical properties μ_s' and μ_a, the reduced scattering coefficient and the absorption coefficient respectively, of a turbid media are given. Then it is well known [1, 6], that the photon density $\Phi(x,t)$ in space and time due to an isotropic source in a highly scattering media can approximately be calculated as solution of the diffusion equation

$$-\nabla_x \cdot \left(\frac{1}{3(\mu_s'(x) + \mu_a(x))} \nabla_x \Phi \right) + \mu_a(x)\Phi + \frac{1}{c}\frac{\partial \Phi}{\partial t} = q(x,t) \quad (1)$$

where $q(x,t)$ stands for a source term and c is the light speed in the media. This is the so called diffusion approximation. Putting

$$\kappa(x) := \frac{1}{3(\mu_s'(x) + \mu_a(x))} \quad (2)$$

as diffusion coefficient for short and assuming a time harmonic point source with frequency $\omega = 2\pi f$

$$q(x,t) = \delta_y(x) e^{\mathrm{i}\omega t} \quad (3)$$

the spatial part $\Phi(x,t) = u(x)e^{i\omega t}$ of equation (1) is described by

$$-\nabla_x \cdot \left(\kappa(x) \nabla_x u \right) + \left(\mu_a(x) + \frac{\mathrm{i}\omega}{\mathrm{c}} \right) u = \delta_y(x) \quad (4)$$

It is this equation which we will refer to as forward model in the following.

3 MEASUREMENTS

The forward model is applied to a bounded domain $\Omega \subset \mathbb{R}^n$ in space. This makes it necessary to add some boundary condition to equation (4). It was experimentally confirmed [7] that a valid approximation is given by

$$u(y) + 2\kappa(y)\frac{\partial u}{\partial \nu} = 0, \quad y \in \partial\Omega \tag{5}$$

where ν denotes the normal vector to the boundary $\partial\Omega$ pointing outwards. For computational simplicity we want to restrict ourselves to the pure Dirichlet boundary condition $u|_{\partial\Omega} = 0$ instead of using equation (5) in the following.

As measurement we want to take the flux rate of photons in direction of the outward normal ν at the boundary,

$$g(y_d) = -\kappa(y)\frac{\partial u}{\partial \nu}\bigg|_{y=y_d} \quad \text{for} \quad y_d \in \partial\Omega \tag{6}$$

Let L be the partial differential operator introduced by equation (4). The mapping $\Lambda : H^{1/2}(\partial\Omega) \longrightarrow H^{1/2}(\partial\Omega)$, $\varphi \mapsto g$ given by (6) where u is the solution of $Lu = 0$, $u|_{\partial\Omega} = \varphi$ is known as Dirichlet-to-Neumann map [3]. Therefore, the detection of scattering and absorption by tomographic means amounts to the recovery both of the diffusion and absorption coefficient in equation (4) from the knowledge of the operator Λ. To this end, we assume that the unkonwn coefficients μ_a, κ are element of some Hilbert space $H(\Omega)$, e.g. $H = L^2$. We set

$$f \equiv f(x) := \Big(\mu_a(x),\, \kappa(x)\Big)^t, \; x \in \Omega \tag{7}$$

and define the residual operators

$$R_j(f) = \left(-\kappa\frac{\partial u_j(f)}{\partial \nu} - g_j\right)\bigg|_{\partial\Omega} \tag{8}$$

for $j = 1, \ldots, p$ if we have p distinct source positions $y_1, \ldots, y_p \in \partial\Omega$. Clearly these operators are nonlinear. Now, the inverse problem by NIR tomography is solved if we can find an element $\hat{f} \in H \times H$ such that

$$R_j(\hat{f}) = 0, \quad j = 1, \ldots, p \tag{9}$$

4 ITERATIVE METHOD

In order to minimize the nonlinear operator

$$\min_f ||R(f)||, \quad R = \Big(R_1, \ldots, R_p\Big)^t \tag{10}$$

we use the iteration

$$f^{k+1} = P(f^k) \quad \text{for} \quad k = 0, 1, \ldots \tag{11}$$

where

$$P(f) = (P_p \circ \ldots \circ P_1)(f) \tag{12}$$
$$P_j(f) = f - R_j'^*(R_j' R_j'^*)^{-1} R_j(f) \tag{13}$$

and $R_j' = R_j'(f)$, $R_j'^* = R_j'(f)^*$ denotes the Fréchet derivative of R_j at the point f and its adjoint, respectively. This iteration can be regarded as a nonlinear extension of the method of Kaczmarz [2, 5]. Unfortunately, the computational burden for determining the operator $(R_j'(f)R_j'(f)^*)^{-1}$ in each step of the iteration is far too high. Therefore, instead of using equation (13) our proposal is to apply the modified equation

$$\tilde{P}_j^{\varrho}(f) = f - \varrho R_j'(f)^* C_j^{-1} R_j(f) \tag{14}$$

where ϱ denotes a relaxation parameter and $C_j = R_j'(f^0)R_j'(f^0)^*$ is a constant operator which we can precalculate. This has to be done carefully because C_j is very ill-conditioned. To build the inverse of C_j we use the truncated singular value decomposition (tSVD). Other methods of regularization could be possible, e.g. Tikhonov-Phillips with regularization parameter γ as indicated in the pseudo code below. Note that C_j depends on the distribution of source and detector positions.

Using Green's formulas the adjoint operator $R_j'(f)^*$ is easily calculated to be given by

$$R_j'(f)^*\varphi = \begin{pmatrix} (\overline{u_j(f)}\, z)(x) \\ (\nabla_x u_j(f) \cdot \nabla_x z)(x) \end{pmatrix} \tag{15}$$

where $x \in \Omega$ and $z(x)$ is the solution to the boundary problem $\overline{L(f)}z = 0$, $z|_{\partial\Omega} = \varphi$.

An algorithm of this method would be as follows.

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Choose an initial
  guess f^(0) = (μ_0, κ_0)^t
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for $j = 1, \ldots, p$ {

  $C_j = R_j'(f^{(0)}) R_j'(f^{(0)})^*$

}

for $k = 1, \ldots, \#\texttt{sweeps}$ {

  $f_0 = f^{(k-1)}$

  for $j = 1, \ldots, p$ {

    $\varphi = -R_j(f_{j-1})$

    $\tilde{\varphi} = (C_j + \gamma I)^{-1} \varphi$

    $f_j = f_{j-1} + \varrho R_j'(f_{j-1})^* \tilde{\varphi}$

  }

  $f^{(k)} = f_p$

}

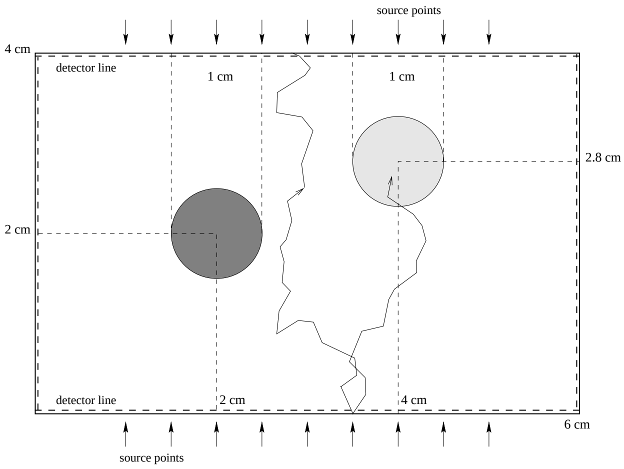

Figure 1: **Geometry**

5 RESULTS

A rectangular domain with dimension 6×4cm is taken as geometrical setup. The optical properties of the surrounding media are chosen to be $\mu_a = 0.025\text{cm}^{-1}$ and $\mu_s' = 12.0\text{cm}^{-1}$. Two spots are placed arbitrarily within the domain. One spot should indicate higher absorption ($5\times$ background value), the other spot is characterized by higher scattering ($3\times$ background value). The diameter of both spots varies from 10mm downto 2mm. As modulation frequence we take 110MHz and assume a light speed of $0.75c_0$ in the media. Using these parameters data sets for 14 source positions at the boundary are calculated by a finite difference scheme. The data is measured at the whole boundary whereas the source positions are equidistantly distributed only on the long sides of the rectangle.

The relaxation parameters are set to $\gamma = 0.2$ and $\varrho = 1.2$, respectively. The computation time was about 10min for each reconstruction with 10 sweeps. The precomputation of the operators C_j took about 40min. As hardware we use an UltraSparc with 143MHz.

The reconstructions are shown in Fig. 2–4. In the first row the original phantom is displayed. The second row shows the reconstruction result after 20 sweeps. The cross section through the original phantom and reconstruction is given in the third row. The last row shows the values of $R_j(f_{j-1})$ for the first sweep and for the tenth sweep, respectively.

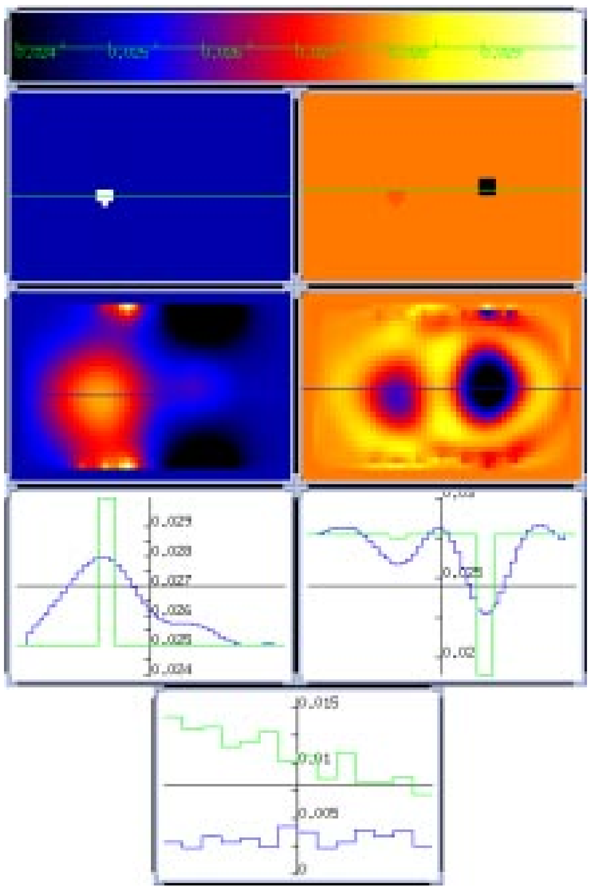

Figure 2: **Reconstruction**

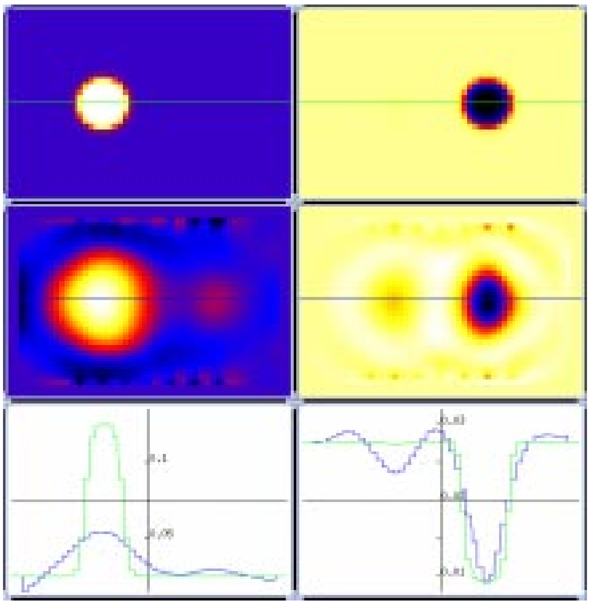

Figure 3: **Reconstruction**

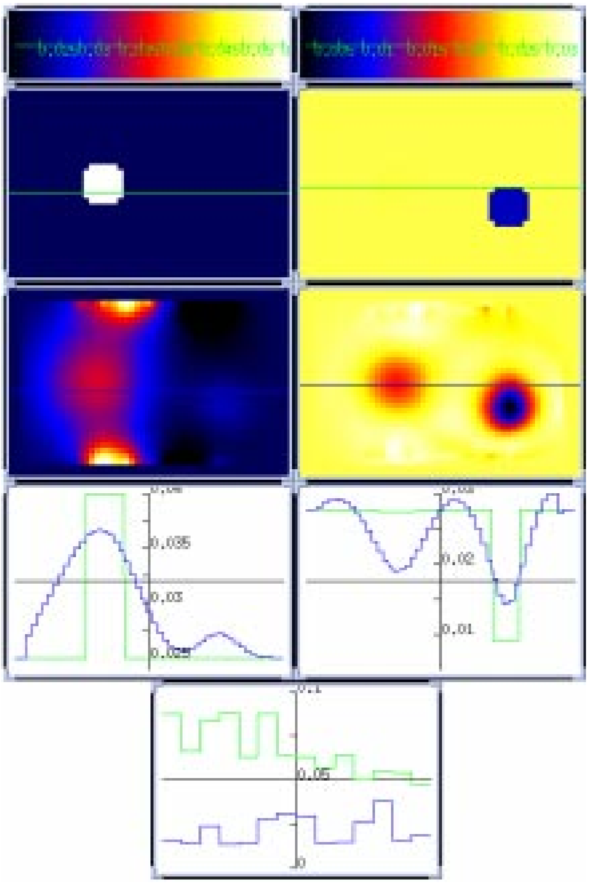

Figure 4: **Reconstruction**

References

[1] Kaltenbach J.P., Kaschke M., Frequency and Time-Domain Modelling of Light Transport in Random Media, in *Medical Optical Tomography: Functional Imaging and Monitoring*, ed. G.Müller, SPIE Optical Engineering Press, Vol. **IS11** (1993), 65–86.

[2] Meyn K.-H., Solution of Underdetermined Nonlinear Equations by Stationary Iteration Methods, *Num. Math.* **42** (1983), 161–172.

[3] Nachman A.I., Reconstructions from boundary measurements, *Ann. Math.* **128** (1988), 531–576.

[4] Natterer F., Numerical Solutions of Bilinear Inverse Problems, *Technical Report* 19/96 N, Fachbereich Mathematik der Universität Münster, 1996.

[5] Natterer F., *The Mathematics of Computerized Tomography*, Wiley, New York, 1986.

[6] Paasschens J., *On the Transmission of Light through Random Media*, PhD thesis, Rijksuniversiteit te Leiden, 1997.

[7] Schweiger M., Arridge S.R., Hiraoka M., Delpy D.T., The finite element method for the propagation of light in scattering media: Boundary and source conditions, *Med. Phys.* **22** (1995), 1–14.