Microwave Imaging of 3D Lossy Dielectric Objects Using Algebraic Reconstruction Techniques

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Abstract *- The Algebraic Reconstruction Technique (ART) is applied in this paper to the image reconstruction of a 3D lossy dielectric object in which microwave diffraction is neglected as an approximation. The data on microwave attenuation are used to reconstruct the images. The images for stacked 2D and complete 3D arrangements are presented showing the usefulness and limitation of the linear reconstruction techniques with and without noise.*

Keywords : Microwave Imaging, Algebraic Reconstruction Technique

1. INTRODUCTION

The use of microwaves for tomographic imaging of a dielectric object has received a great attention in the recent years as microwave tomographic systems may be used as an alternative to x-ray or γ-ray systems. Compared with x-rays or γ-rays, microwave radiation is non-ionising providing a safer technique for producing quality images. However, unlike x-ray or γ-ray systems producing images of attenuation, which depending only on the object density [1], microwave tomographic systems provide images of dielectric constant, or loss tangent, which is often frequency and temperature dependent.

Microwave tomographic imaging can be made by illuminating object with a microwave field at different angles and measuring the scattered fields produced by the object at each illumination, with the use of transmitting and receiving antennas [2]. The image of the object is obtained by making use of the data of the scattered fields and an image reconstruction algorithm. The reconstruction algorithm has to take into account the diffraction inside object. As a consequence of the diffraction, the image reconstruction for microwave systems is non linear and the algorithm is in general more complicated than that for a x-ray or γ-ray system [2]. The computation for the non-linear inverse problem is much heavier, and the solution is sensitive to the selection of initial parameters as a result of the ill-possedness of the inverse problem [3]. However, for lossy dielectric objects, the effect of diffraction inside the object can be neglected so that linear reconstruction techniques such as Algebraic Reconstruction Technique (ART) can be used to solve the inverse problem. This has been successfully demonstrated for a 2D object [4]. In this paper, the ART is applied to the reconstruction of a 3D lossy dielectric object with the measurement of microwave attenuation, and the use of ray width to represent the microwave transmission as for x-rays or γ-rays. The 3D object is first treated as a 2D object with 5 layers with transmitting and receiving antennas placed on the each layer, and then as a complete 3D object with antennas placed on selected layers around the object. The images for an arm model will be presented for both two-dimensional and three dimensional configurations. The results on the effect of the noise and ray width will also be presented.

2. THEORY

Consider a cylindrical object of an arbitrary cross-section surrounded by a homogeneous medium, such as water, in which microwave transmitting and receiving antennas are placed circumferencially around the object as shown in Figure 1. Assuming the dielectric constant of the object does not differ very much from the exterior homogeneous medium and antennas are matched to the medium in term of impedance, the microwave transmission loss across the object will mainly depend on the loss tangent along the direct propagation path. For one particular transmission from A_1 to B_1 as shown in Figure 1, the total measurable transmission loss along the propagation path can be written as

$$P = \int \alpha . dl \qquad (1)$$

where α is the attenuation constant.

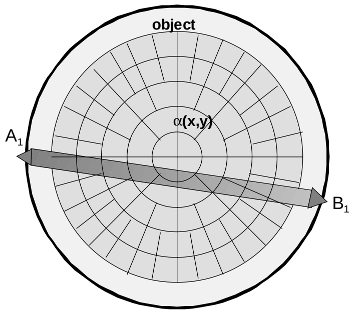

Figure 1: A cross sectional view of an object under imaging with a microwave transmission from A_1 to B_1

As a numerical approximation, the object is divided into *N* cells of identical areas. In each cell, the attenuation constant $\alpha(x,y)$ is also assumed to be the same. At the j^{th} cell, $\alpha(x_j,y_j) = \alpha_j$. Since microwaves can propagate along the path with a limited width, which is referred as a ray width in ray terms, the total attenuation given in (1) would need to take into account the attenuation constants not only at those cells on the path, but also at other cells within a defined ray width. The line integral in (1) is then referred as a raysum. The raysum of the i^{th} ray with a ray width τ, as illustrated in the shaded path in Figure 1 for a 2D object, is given by

$$\sum_{j=1}^{N} w_{ij}\alpha_j = p_i \quad where \quad i = 1..M \qquad (2)$$

and w_{ij} is the weighting factor for α_j with the incidence of the i^{th} ray. The weighting factor w_{ij} can be taken to be unity when the cell is inside the width of the i^{th} ray or a fraction of the area of the j^{th} cell inside the raywidth

To solve (2) for attenuation constants, the iterative method proposed by Kaczmarz [5] can be used. Defining $\vec{w}_i = (w_{i,1}, w_{i,2},\ldots w_{i,n})$, and $\vec{w}_i.\vec{w}_i$ as the dot product of $\vec{w}_i$ with itself, the iterative equation for α is given by [6]

$$\vec{\alpha}^{k+1} = \vec{\alpha}^k + \frac{p_i - \vec{\alpha}^k.\vec{w}_i}{\vec{w}_i.\vec{w}_i}.\vec{w}_i. \qquad (3)$$

or

$$\alpha_j^{k+1} = \alpha_j^k + \frac{p_i - q_i}{\sum_{k=1}^{N} w_{ik}^2}.w_{ij}. \qquad (4)$$

where

$$q_i = \sum_{j=1}^{N} w_{ij}\alpha^k{}_j$$

and $k=0,1,2 \ldots, K$, for the computation of the attenuation constant at each cell with an initial guessed value of α_j for j=1,2..N. For simplicity w_{ij} in (3) and (4) is taken to be

$$w_{ij} = \begin{cases} 1 \rightarrow & \text{if the cell is inside the raywidth} \\ 0 \rightarrow & \text{otherwise} \end{cases}$$

This can be applied to both 2D and 3D objects. However, for 3D object, the ray width represents the diameter of the ray extending in three dimensions.

The convergence of the iteration in (4) is monitored by using the residue error function

$$ErrQ^{k+1} = \frac{\|Q^{k+1} - P_0\|}{\|P_0\|} \qquad (5)$$

where

$$Q^{k+1} = \sum_{i=1}^{M}\sum_{j=1}^{N} w_{ij}\alpha_j^{k+1} \qquad (6)$$

and P is the summation of all measured raysums. The iteration process will end when the condition

$$Qerr = \left|ErrQ^{k+1} - ErrQ^k\right| < 10^{-4} \qquad (7)$$

is satisfied or the maximum number of iteration is reached.

In the case of simulation, the value of attenuation constant at each cell, denoted by $\alpha_{o,j}$, can be set for the solution of measured raysums. The error of the reconstruction can then be evaluated against each iteration using the error function

$$Err\alpha^{k+1} = \frac{\|\alpha^{k+1} - \alpha_0\|}{\|\alpha_0\|} \qquad (8)$$

The stability of the iteration process can be tested by introducing a noise to the measured raysums, with the signal-to-noise ratio defined as in [7] as follows

$$SNR = 10\, log_{10} \frac{\|P_0\|}{\|noise\|} dB \qquad (9)$$

This iteration process will be used to reconstruct a 3D object as described below.

3. THREE DIMENSIONAL MODEL

To test the iteration process described in Section 2, a 3D cylindrical object is used, as shown in Figure 2, which is referred as the arm model. This model is commonly used for biomedical microwave imaging as in [2][3][7]. The object has a diameter of 12cm and a height of 2.3cm. It simulates a human arm with dielectric properties describing the bones, muscles and fat. The object consists of 5 layers, on each of which the dielectric constant has a different distribution. The object is placed inside water. The system is considered to operate at 2.454 GHz. The dielectric properties of bone, muscle, fat and water at this frequency is tabulated in Table 1. The attenuation constants in these materials can be calculated using

$$\alpha = Re\left[\omega\sqrt{\mu_o \varepsilon_o (\varepsilon_r' - j\varepsilon_r'')}\right] \tag{10}$$

which is given in Table 1.

Material	ε_r'	ε_r''	α (Neper/cm)
Bone	8.0	1.32	1.45
Muscle	49.6	16.5	3.66
Fat	4.5	0.84	1.09
Water	77.3	8.66	4.52

Table 1: Dielectric properties of bone, muscle, fat and water at 2.454GHz

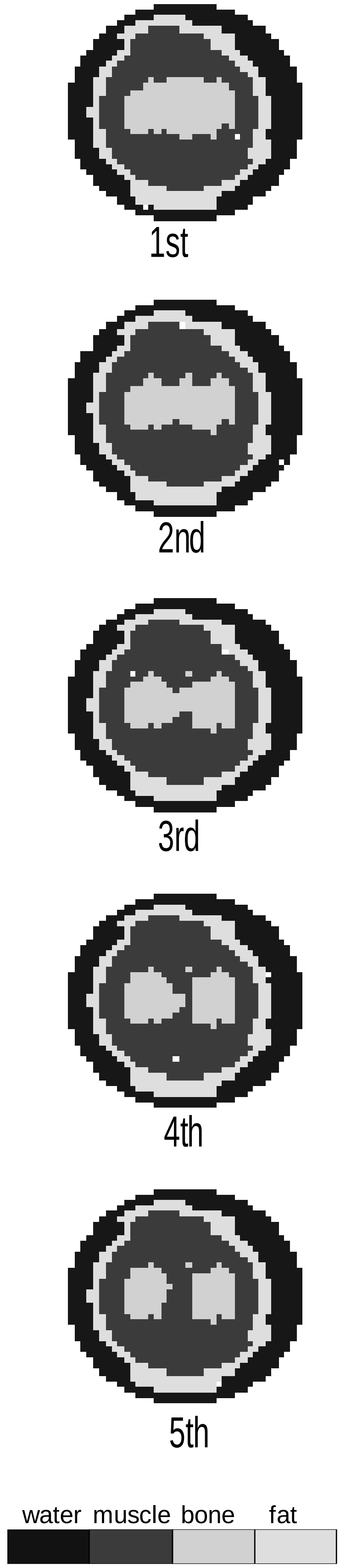

Figure 2: Cross-sections of the five layered 3D arm model

4. NUMERICAL RESULTS

4.1 Two-dimensional Reconstruction

For the two-dimensional reconstruction, the first layer of the object is considered, which is treated as a 2D object. A total of 32 antennas which are capable of transmitting and receiving are placed, with equal spacing, around the object at a radius of 8cm as shown in Figure 3. For simulation purpose, the cross-section is divided into 484 cells. The measured data at 32 antenna positions with all of them also transmitting in turn, which are needed for the iterative reconstruction process in (4), are calculated with the assigned attenuation constants at 2.454GHz as given in Table 1 for different types of tissues inside the object. For different ray widths used ranging from 1mm to 19mm with initial values 0 for all cells, the reconstructed images are shown in Figure 4. The error of reconstruction against the ray width after 200 iterations is shown in Figure 5.

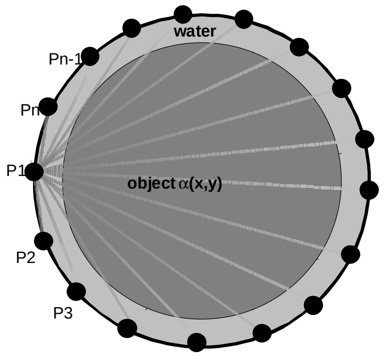

Figure 3: Antenna arrangement for two-dimensional reconstruction

It can be seen that the two-dimensional reconstruction using the ART algorithm can provide good images. Also, the reconstruction does not depend very much on choice of ray width. This is very useful as the ray width for microwave transmission can hardly be defined precisely as for x-rays or γ-rays. An error of 6% can be achieved using a ray width comparable to the size of the cell.

In the above reconstruction, the measured data do not contain any error. However, in practice, the measured data are subject to the effect of noise and measurements errors. The effect of noise to the reconstruction can be seen from the images shown in Figure 6, with SNR=50, 40, and 30 respectively using ray width of 10mm, after 25 iterations.

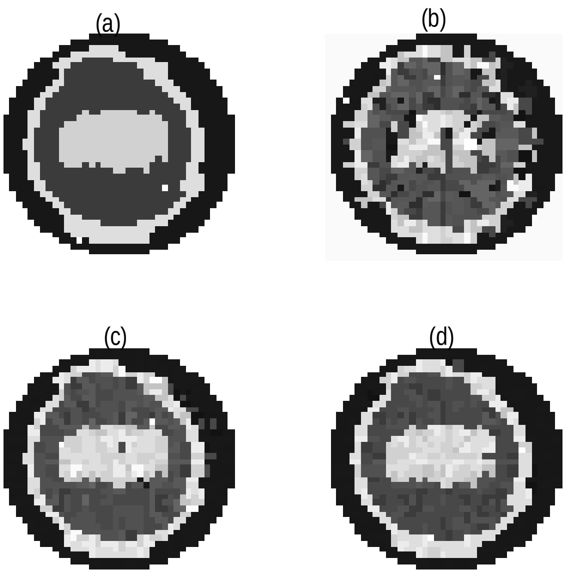

Figure 4: The result of reconstruction first layer arm model by using ART algorithm using different ray widths a) exact value b) 1mm c) 5mm and d) 10mm.

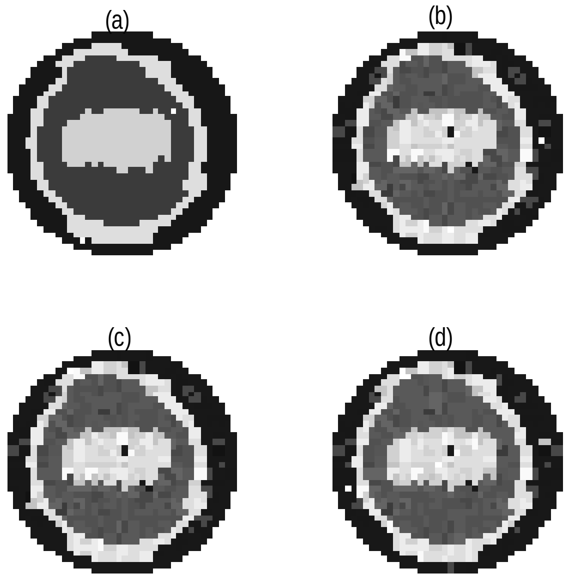

Figure 6: The reconstruction images of the first layer of the arm model(a) by using ART algorithm with a ray width 10mm with noise and SNR of b) 50dB c) 40dB and d) 30dB.

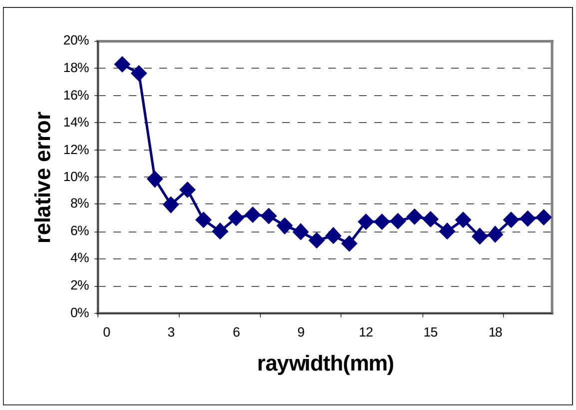

Figure 5: Relative error against ray width after 200 iterations using two dimensional algorithm.

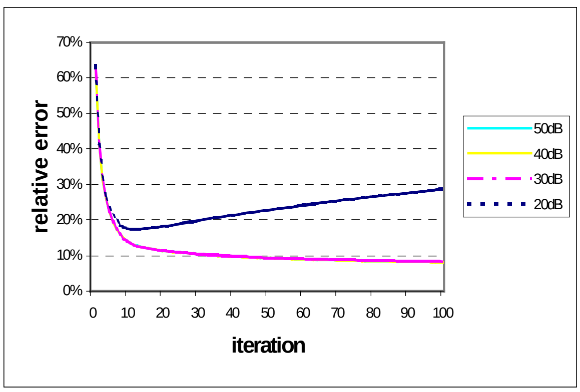

Figure 7: Relative error of reconstruction the first layer of the arm model with SNR of 50 dB, 40 dB, 30 dB, and 20 dB against iteration numbers

The error increases as the signal-to-noise ratio decreases, as shown in Figure 7 against the number of iteration for a ray width of 10mm. The errors are below 10% for SNR > 30dB with 25 iterations. However, for SNR=20dB, the algorithm fails to produce accurate images.

Extending the two-dimensional reconstruction to other four layers of the object with 32 antennas placed on the layer of interest. The results are shown and compared in Figure 8 with different ray width and Figure 9 with noises. Images of similar quality to the first layer can be obtained for the 5-layered 3D object.

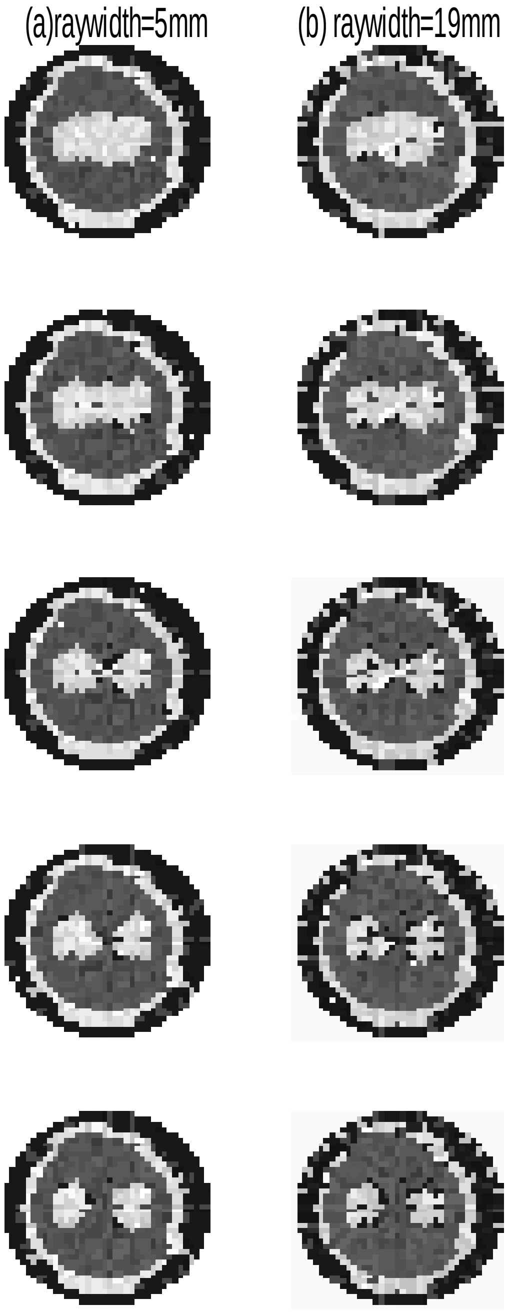

Figure 8 The reconstructed images of the five-layered 3D arm model by using two dimensional ART algorithm on each layer with different ray widths.

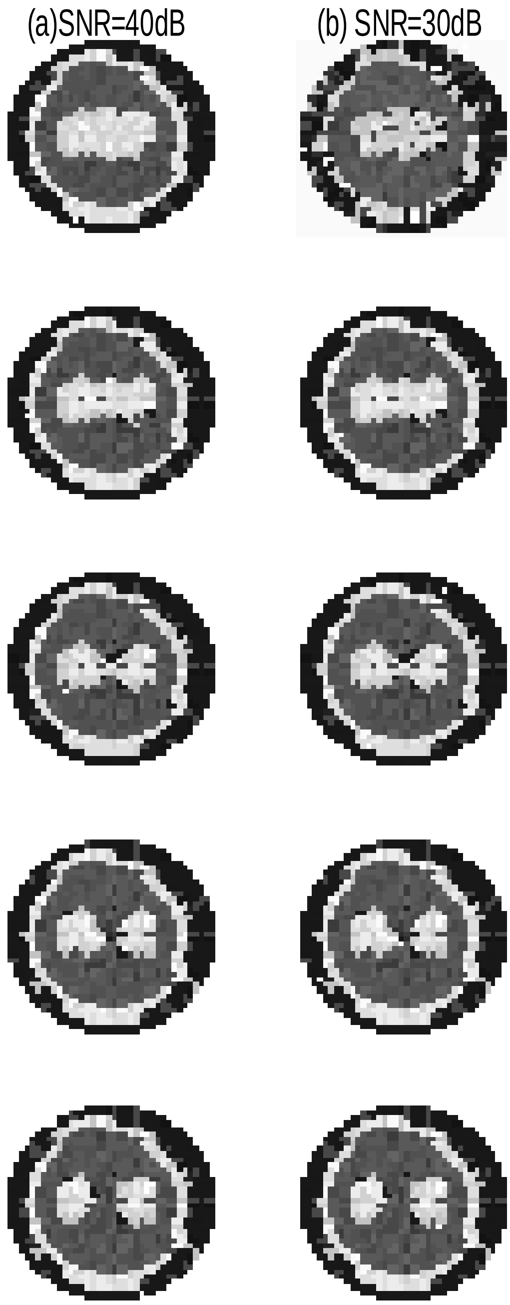

Figure 9 The reconstructed images of the five-layered 3D arm model by using two dimensional ART algorithm on each layer with ray width of 5mm with noise SNR of a) 40dB and b) 30dB.