

An algorithm based on Perturbation Theory for Electrical Capacitance Tomography

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Abstract - This paper presents a new algorithm for an image reconstruction for Electrical Capacitance Tomography (ECT). This algorithm is based on the perturbation theory and uses an estimate of the permittivity distribution from a back-projection algorithm. By providing a guess of the ϵ distribution, the problem of limiting the degree of freedom or uncertainty in the image-reconstruction process will be solved. The gain of this new method would not only be a higher speed for the imaging process as it would be possible to implement it on a parallel computation device, in this case the Cellular Neural Network (CNN) paradigm, but also a better resolution than usual back-projection or other techniques.

Keywords: Reconstruction algorithm, perturbation theory, ECT, Cellular Neural Network

1. INTRODUCTION

This paper presents an attempt to develop a new algorithm for image reconstruction in Electrical Capacitance Tomography (ECT). The main objective is to provide a quicker imaging system by porting the new algorithm to a Cellular Neural Network (CNN) board. The CNN paradigm [1] [2] [3] is based on a highly connected mesh of cells that is designed to perform only neighbourhood operations. This paradigm has many applications in the field of image processing, and PDE solving is only one of other applications.

To solve an inverse problem for capacitance tomography, a back-projection algorithm was applied by Huang et al., (1988) [4]. It was assumed that the effect of the dielectric distribution on the sensitivity distributions of the sensors is insignificant, and the sensitivity distribution in the sensing area is uniform. To improve the image quality Xie et al., (1992) [5] calculated the values of sensitivity matrices. These were calculated assuming that in only one pixel material permittivity and in the rest. In a back-projection algorithm the grey level approximating the distribution of permittivity is obtained by multiplication of a vector representing the measurements taken from a multi-electrode system, by a pre-calculated sensitivity matrices. Therefore, a back-projection algorithm is just a simple vector matrix

multiplication. However, the sensitivity matrix is actually dependent on the unknown distribution due to the non-linear relationship between the measured capacitances and the permittivity distribution. Therefore, the application of the pre-calculated matrices implies the linearisation of the non-linear problem.

Recently, a quantitative algorithm has been developed by Yang et al., (1997) [6]. This algorithm, iterative in its character, is based on solving a forward problem (capacitances are calculated for a given material distribution). At each iteration the sets of errors between the measured capacitances and calculated capacitances is used, via the back-projection algorithm, to reconstruct an error image by means the sensitivity matrices, which are used in the back-projection algorithm. The error image represents, to some qualitatively trustworthy extent, the difference between the current image and true image. It is added to the current image to obtain a new image. The process is repeated until a satisfactory image is obtained.

Finite Element Methods (FEM) are widely used to solve partial differential equations of various kinds; like acoustics, structural mechanics and electromagnetism. FEM methods often involve several matrix inversions at some stage in the case of ECT; it is well known that those matrices are strongly ill-conditioned. Methods like singular value decomposition or least squares and their

derivatives are usually used to provide estimates that vary greatly in quality. In this paper, a specific approach is presented and tries to avoid the error generation or the loss of resolution usually associated with current methods.

2. CONCEPT AND IDEA

The algorithm presented here is based on perturbation theory. By assuming that we already possess an estimate of the permittivity distribution ϵ , it is expected that this first estimate or guess can be updated to match the capacitance readings on different electrode-pairs during the scanning process. According to an information theory point of view, the imaging method definitively lacks in information content. The readings are made on the periphery of the domain under consideration and the expected result is on the whole mesh covering the circular slice of the tube under consideration. The concept of providing a reconstruction algorithm with a first approximation lies in the expectation of limiting the number of degree of freedom associated with the solution of a partial differential equation with its source of information limited to boundary values.

To uphold this idea of dealing with a lack of information, the main hypothesis made is on the size of the variations allowed to take place within the system. The variations on the permittivity distribution and the associated potential distributions are considered very small when compared with their previous values in the algorithm as expressed in (1).

$$(1) \quad \begin{aligned} V &= V_0 + dV \\ \epsilon &= \epsilon_0 + d\epsilon \end{aligned}$$

Introducing these considerations in the governing PDE (2) leads to an approximation for a new value for ϵ , the permittivity distribution. The updated value is given in the next paragraph after the definition of the different notations.

$$(2) \quad \nabla \epsilon \cdot \nabla V = 0$$

The algorithm is based on a loop structure, when the error between the capacitance measurements and the capacitance evaluations falls below a certain threshold, the current permittivity distribution is considered as the final image of the system.

3. ALGORITHM AND IMPLEMENTATION

The algorithm can be described in successive steps. The notations previously introduced are in use. The implementation of the approach has been developed and tested on the MATLAB ®

Version 5 [7] for Windows™ environment with the Partial Differential Equation (PDE) Toolbox.

3.1. Algorithm description

- From a given permittivity distribution ϵ , eight potential distributions are computed in turn. Each distribution is the result of one electrode being set at a 15V potential or "fired" whilst the others are set to 0V or "grounded". These potential distributions are called the V_i , with the index i varying from 1 to 8. The governing equation for each firing is (3):

$$(3) \quad \nabla \epsilon \cdot \nabla V_i = 0$$

- From the V_i distributions, the capacitance C_{ij} ($i \neq j$) are computed. C_{ij} is the value of the capacitance between electrodes i and j when i is fired.
- The C_{ij} are then compared to the C_{mij} , capacitance measurements on the actual laboratory prototype. The difference in value reflects the difference between what it is expected to be seen, or the initial guess, and the effect of the actual distribution. The capacitance difference translates in a potential flux that can be seen as a set of Neumann boundary conditions on the electrodes j .
- The Neumann boundary conditions from the previous step are combined with one Dirichlet condition $V=0$ on the electrode i , which is fired, to form the set of boundary conditions for the perturbation potentials. Figure 1 holds for electrode 5 being grounded and its corresponding errors on the other electrode being set as Neumann conditions. V_{p5} stands for the perturbation potential associated with the firing of electrode 5. It is computed from the error generated by the difference of capacitance value between the capacitance values estimated from V_5 or C_{5j} , in the first step, and the capacitance measurements C_{m5j} collected when electrode 5 is fired. n is the standard notation for the normal to the electrode under consideration. When diffusing the error, ϵ is considered as constant, its effects are already included in the error estimates as the V_i potentials already reflect a non-uniform distribution.

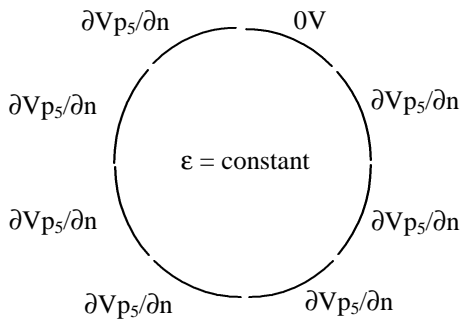


Figure 1: Neumann conditions for electrode 5 grounded

After those steps, the values deemed necessary to update the permittivity distribution are available. The following notations apply:

- V_i Potential distributions obtained from a given distribution ϵ .
- V_{p_i} Potential distributions obtained from the diffusion of the Neumann boundary conditions corresponding to the error field characterising the firing of electrode i .
- ϵ^{old} Permittivity distribution from the previous step in the loop, or first guess in the case of the first loop.
- ϵ^{new} Permittivity distribution update.
- γ Correction factor. The value is determined to insure the hypothesis set in (1) are abided by.

The new quantity of the permittivity distribution can then be assessed in (4). This ends the general description of the main algorithm.

$$(4) \quad \mathbf{e}_i^{new} = \mathbf{e}^{old} - \mathbf{g} \frac{\nabla \cdot (\mathbf{e}^{old} \nabla V_{p_i})}{\nabla^2 V_i}$$

3.2. Software implementation

The MATLAB ® environment has been chosen for its set of pre-built functions and facilities to develop quickly and reliably an application corresponding to the approach on ECT imaging. The additional toolbox on PDEs has provided the mesh generation algorithm and some new functions or adaptations have been developed to cope with the specificity of the algorithm.

The first method used to solve the PDE associated with the problem at hand was the Finite Element Method (FEM). In-built functions provided by the development environment have

allowed a rapid implementation to take place. However considerations about the validity of the matrices being inverted rapidly came to light as the first results appeared strongly unstable. Because of the size of the mesh and the relatively small number of pieces of information available the FEM method assembled ill-conditioned matrices, thus rendering the solutions for potential distributions all the more questionable. MATLAB ® functions have then been written to implement a Finite Difference (FD) approach to the diffusion problem. Considering the stability qualities of this kind of approach, a better, more stable and reliable, way of finding the potential distribution has been implemented.

To counteract strong variations in the value of laplacian and gradient estimations close to the electrode being fired, several adjustments have been made in the development of equation (4). This adjustments being related to the assumptions that lead to (4) involved only the computation of some residual value from laplacian related to the solving of (3) and the likes. They are not essential to the description of the method. Testing of the different components of the program independently from the main algorithm have shown that the application was the closest possible translation of the idea and the results discussed in the next paragraph can be seen as a good interpretation of the proposed concept.

4. RESULTS AND EXPECTATIONS

This section presents the results obtained by the simulation and attempts to draw implementation related conclusions, the general findings linked with the nature of the new approach are developed further on. The outcome not being of the required quality, a second subsection discusses what was expected from the proposed new approach.

4.1. Visual representation

Figure 2 shows the original mesh and the permittivity distribution given as a first guess for a particular simulation. The working example was to give the same object shape with a minor difference in the value of the object permittivity, in this case the guess was at $\epsilon=1.9\epsilon_0$ and the original simulation was done for a value of $\epsilon=2\epsilon_0$.

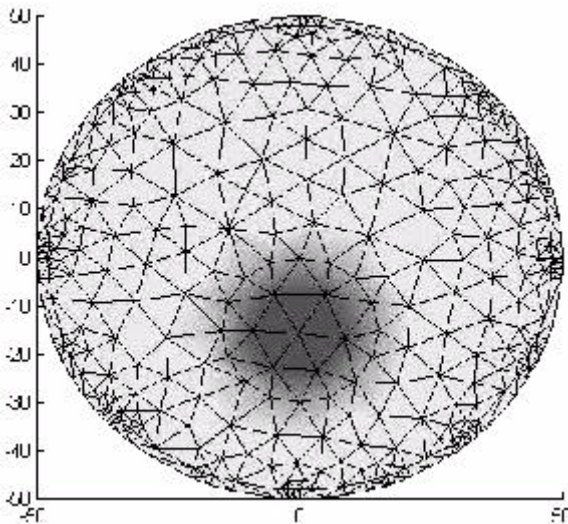


Figure 2: First approximation for the permittivity distribution

The next figure shows the result of updating the permittivity distribution after one step in the algorithm. It is clear that Figure 3 does not give the expected result of a slight variation in value and geometry bound to appear according to the primary assumptions and considerations made earlier. After a few steps, the ϵ quantity at the centre of gravity of the mesh triangles becomes totally meaningless, some thousand times too great in absolute value; it is oscillating strongly and diverging even more step after step. In Figure 3, the colour bar values are scaled by a factor ϵ_0 to suppress the dimension.

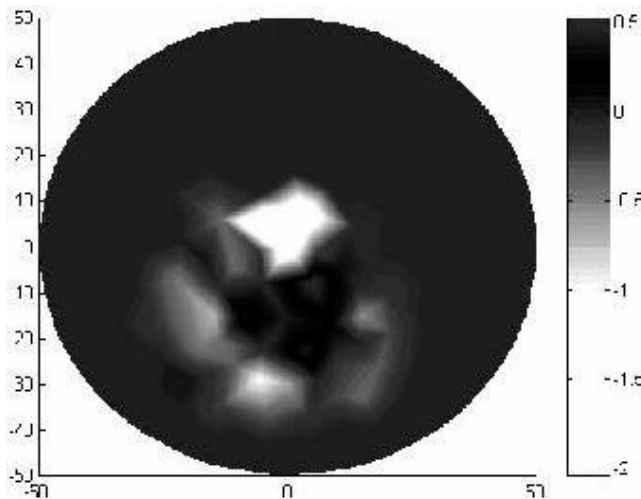


Figure 3: Result after one step

4.2. Expected result

A perfect update of the first approximation could never have been the outcome expected from this new approach. The aim was to provide an algorithm for a quicker imaging system based on parallel computing and also to improve the resolution of current ECT systems. Some discrepancies were expected especially in the

shape of the object as the resolution is related to the number of electrodes, but the aim of the simulation was to ascertain the possibility of getting a reconstructed image by only small and repetitive updates; on this point it has failed. However, by observing the results of the different simulations, problems linked with consistency in the field of information processing have raised other issues that are likely to open another avenue for a slightly different approach.

5. FUTURE DEVELOPMENTS

From the early stages, the proposed approach has always relied upon the solution of the forward problem by expressed equation (2). The reason for this is that this equation can be solved by parallel means, especially with the Cellular Neural Network (CNN) paradigm. Even though the approach so far has proven unsuccessful, it is believed that by developing an image reconstruction algorithm which can be easily translated to a parallel architecture, new speed level in tomographic imaging can be reached. Taking into account that observation and keeping the concept of perturbation techniques in mind, the research is now geared towards examining the effect of a minimal variation in a small number of points in the permittivity distribution. By restricting the number of updates to the available number of pieces of information gathered at every scanning of the domain under consideration, it is expected not to generate error as previously observed in the attempt described in this paper.

This approach requires solving the same PDE as before and would be likely to be implemented on specialised hardware, such as a CNN array, if the simulation on a conventional development platform is successful.

6. CONCLUSIONS

From the start, it was known that by having only eight sources of information at a time, each electrode reading, the approach was supposed to generate some error. The objective was that by providing an approximation of the permittivity distribution, the gap in information content would be bridged. It is not so, and the reason for that is likely to be related to the properties related to diffusion problems. When each set of boundary conditions, on the periphery of the domain, is diffused towards the centre to give an image of a potential distribution, a concept of negative exponential dispersion is at work. Without delving into mathematical considerations, it seems obvious, with the benefit of hindsight, that the process described here had tried to reverse the trend of the diffusion concept by trying to update a parameter acting on the potential

generation. In the end, instead of generating information, error was generated and on a large scale.

This general observation and several others, more closely linked with algorithm development and implementation, have provided some guidance in the development of the next stage of the implementation of a new algorithm for the development of a new ECT imaging technique.

REFERENCES

- [1] L.O. Chua and L. Yang "Cellular Neural Networks: Theory", IEEE Transactions on Circuits and Systems, vol. 35, pp 1257-1272, October 1988.
- [2] J.A. Nossek and T. Roska "Guest Editors' Introduction", IEEE Transactions on Circuits and Systems-I, vol. 40, March 1993.
- [3] V. Cimagalli and M. Balsi "Cellular Neural Networks: A Review", Proceedings of the Sixth Italian Workshop on Parallel Architectures and Neural Networks. World Scientific. Vietri sul Mare, Italy, May 12-14, 1993.
- [4] X.M. Huang, T. Dyakowski, A.B. Plaskowski and M.S. Beck. "A tomographic flow imaging system based on capacitance measuring techniques", Proceedings of the ninth International Conference on Pattern Recognition, Rome, Italy, 1988.
- [5] C.G. Xie, S.M. Huang, B.S. Hoyle, R. Thorn, C.P. Lenn, D. Snowden and M.S. Beck, "Electrical capacitance tomography for flow imaging: system model for development of image reconstruction algorithms and design of primary sensors", IEE Proc. G, 139, pp 89-98, 1992.
- [6] W.Q. Yang, J.C. Gamio and M.S. Beck, "A fast iterative image reconstruction algorithm for capacitance tomography", Sensors and their Applications VIII, Editions A.T. Augousti and N.M. White, IOP Publisher Ltd, pp 47-52, 1997.
- [7] MATLAB, "The Language of Computing", The MatWorks, Inc. 24 Prime Park Way, Natick, MA 01760-1500.